Errors as springboards for remediation of Year 7 students subtraction knowledge

S. Dole and T.J. Cooper

Centre for Mathematics and Science Education,

QUT, Kelvin Grove, Australia.

Abstract

Many errors in arithmetical computation are systematic; they are learned and have become habitual. This paper investigates two methods of instruction for correcting systematic errors and promoting knowledge growth for the subtraction algorithm in upper primary students: (1) structured reteaching, linking symbolic procedures to concrete/pictorial representations; and (2) the Old Way/New Way (O/N) technique, based on proactive inhibition. O/N was successful in changing computational knowledge expediently and fairly effortlessly while the conventional approach proved less successful.

When appropriate knowledge is constructed, initial learning is generally of little concern to teachers/educators. However, when initial learning results in the development of erroneous knowledge, instruction takes on a remedial focus in an effort to change that knowledge. The provision of remediation programs for students experiencing difficulty with the study of mathematics is often a frustrating task. The success of the remediation program is dependent upon many factors including the educator's experience, the student's prior experience of learning failure, the nature of the learning difficulty, the accuracy of the error diagnosis, the relationship between the student and the educator, and the degree of transfer of learning from the corrective setting to the regular classroom (e.g. Ashlock, 1986, Covington, 1985; Dole, 1992).

Within the discipline of mathematics, computational errors often are the first signal that a student is experiencing difficulty. Analysis of errors in computation has revealed that many student errors are not careless or random, but occur regularly and consistently (Brumfield and Moore, 1985; Cox, 1975) and through repetition, have become learned habits. They are produced automatically in response to a stimulus, and in contrast to random, careless errors, are not self-detected nor self-corrected. They are conceptual and learned (Ashlock, 1992). These conceptual, learned errors indicate that the child is capable of learning; that what has been learned, simply, is an incorrect way of doing things (Lyndon, 1989).

For the remediation of systematic errors, approaches incorporating the close linkage of the written representation with the concrete/pictorial representation have been suggested (e.g. Ashlock, 1992; Booker, Irons, and Jones, 1980; Resnick, 1982). Such approaches are based on slow and progressive reteaching in an effort to 'fill the gaps' in the students' knowledge which are regarded as attributable to the error pattern development (Ashlock, 1992; Booker, Irons, & Jones, 1980; Jones & Charlton, 1992; Wilson, 1976; Vallet, 1976). Such programs are based on an 'absence of knowledge' perspective. However, studies incorporating such methods have revealed that students revert back to their erroneous methods despite the intensity of remediation, and that the lack of positive transfer of new learning and display of avoidance behaviour by the students towards corrective instruction are factors affecting knowledge growth (e.g. Bourke, 1980; Wells, 1982; Wilson, 1982). Thus, some students appear to make satisfactory progress under closely supervised and individualised instruction, but these gains do not transfer to the regular classroom. Although improvement may occur in the short term, these gains appear to fade over time (Read, 1987).

The existence of learned errors has implications for corrective attempts in that, despite intensive instructional intervention, many students revert back to their erroneous methods. Lyndon (1989) has proposed that this observed lack of learning transfer and associated regression to erroneous patterns is due to the mental phenomenon of proactive inhibition (PI). Proactive inhibition is an information protection mechanism which is activated when new learning conflicts with prior learning (Underwood, 1966). According to Lyndon, (1989) it is proactive inhibition which actually inhibits knowledge change, and is responsible for the recurrence of error patterns despite intensive remediation. Also, avoidance behaviours are a consequence of the activity of proactive inhibition. Recurrence of systematic errors reinforce in learners their feelings of failure. Avoidance of behaviours are exhibited as a response to avoiding the situation: the best way to avoid failure is never to tryanything new. As stated by James (1980) 'With no attempt there is no failure, and with no failure there is no humiliation.'

Briefly, Lyndon has argued that consistent errors are protected by proactive inhibitions and that PI is actually triggered by conventional remediation methods, evidenced by students exhibiting such behaviours as slowness to respond, an apathetic attitude to the task, frustration, and avoidance behaviours. For effective remediation, Lyndon has contended, the remediator must acknowledge PI as an inhibitor of knowledge change and growth, and as such, remediation programs must be structured to effectively deal with proactive inhibition. Lyndon (1989) has proposed that an alternative approach, termed Old Way/New Way (O/N), deals with such difficulties in remediation. This approach is built around the perspective that the error patterns indicate the presence of knowledge, rather than its absence, and suggests that remediation programs must confront a student's prior knowledge in order to affect change. Applying this approach to remediation of systematic computational subtraction errors, appears to be a 'top-down' approach where erroneous procedures and algorithms are the first point of focus and students are taught to identify their old method and replace it with a 'new' correct method in contrast to conventional remediation where typically procedures are retaught following good 'first-time' teaching models. Lyndon has contended that: (1) the O/N technique bypasses proactive inhibition and enables the remediator to change the child's knowledge base rapidly and permanently; (2) the more or less instantaneous success the child experiences after one trial of the O/N procedure ensures that avoidance learning behaviours are soon eliminated; and (3) the O/N technique enables confidence in ability to learn to be restored.

This paper describes a study (Dole, 1992) in which two methods of remediation were utilised in an attempt to change Year 7 students' (age 12-13 years) erroneous computational subtraction knowledge. The two remediation methods were structured reteaching, a conventional sequence based primarily on suggestions by Booker, Irons, & Jones (1980), and Old Way/New Way. Subtraction performance was measured in terms of Leinhardt's (1988) four knowledge types: (1) intuitive knowledge, the 'everyday' or real world application knowledge, normally acquired before formal instruction; (2) concrete knowledge, the knowledge associated with representation by appropriate concrete materials during instruction; (3) computational knowledge, the 'this is how to do it' knowledge associated with formal procedures; and (4) principled-conceptual knowledge, the "underlying knowledge of mathematics from which the constraints can be deduced." (p. 122).

Method

Subjects

The subjects for the study were sixteen Year 7 students selected upon demonstration of systematic errors in subtraction computation from a pool of sixty Year 7 students attending a suburban Brisbane primary school.

Instruments

The instruments used were a diagnostic error analysis test used to select the students and an interview schedule used to determine students' subtraction knowledge. The researcher-made diagnostic error analysis test consisted of five types of subtraction problems classified according to level of computation skill required (see Table 1). The test contained five problems from each skill level of computational skill, presented in random order. Test performance was scored by examining errors for the existence of a pattern. For any given skill level, a systematic error was defined as one which occurred three or more times out of five attempts (Cox, 1975).

Table 1: *Subtraction algorithm levels of diagnostic test instrument*

Skill	*Example*
Two-digit - two-digits, renaming tens to ones	*53 - 14*
Three-digits - two-digits, renaming from hundreds & tens	*523 - 78*
Three-digits - three-digits, renaming from tens & zero in ones	*260 - 156*
Three-digits - three-digits, renaming from hundreds & zero in tens	*608 - 135*
Three-digits - three-digits, renaming from hundreds & tens and zero in tens	*302 - 158*

The schedule for the structured clinical interviews (Ginsburg, 1981) contained items relating to concrete, intuitive, and principled-conceptual subtraction knowledge (results from the diagnostic error analysis test enabled computational knowledge to be determined). For concrete knowledge, subjects were required to perform the subtraction algorithm using Base 10 (MAB) blocks. For intuitive knowledge, subjects had to solve a real world subtraction problem and create a real world subtraction problem. For principled-conceptual knowledge, subjects had to demonstrate the following understandings: (1) subtraction makes smaller, that increasing the minuend and subtrahend increases and decreases the solution respectively; (2) addition is the inverse of subtraction; (3) the renaming process in decomposition subtraction algorithms; and (4) the role of place value in decomposition subtraction algorithms.

Procedure

Sixty year seven students were given the diagnostic error analysis test. Sixteen students exhibited consistent error patterns, eight in all skill levels of subtraction algorithms, and eight in the three hardest of the five skill levels. The students were randomly assigned to treatment groups 1 and 2, with four students with errors in all skill levels and four students with errors in the three highest levels in each group.

The students in each group were interviewed prior to treatment (the pre-interview). After this, the two groups were withdrawn and given different remediation treatments by the same teacher over 10 consecutive days. Group 1 (structured reteaching) received a series of ten 20-minute lessons, using bundling sticks for initial regrouping activities, using MAB to progress through the five skill levels and using numeral expanders to reinforce regrouping processes for large numbers. Each student's progress was documented and each lesson was contingent upon progress in the previous lesson (as in a teaching experiment, Kantowski, 1978). Group 2 (O/N) received one 10-minute O/N learning trial for the most difficult level, where: (1) the students' errors were activated and labelled 'old way'; (2) an alternative correct method was offered and labelled 'new way'; (3) the students performed five computations the old and new ways; and (4) the students performed 6 computations the new way. This was followed by one 10-minute practice session. Finally, all students were interviewed after the treatment (the post-interview).

Results

The students interview responses were categorised as satisfactory and unsatisfactory with respect to the four knowledge types. Field notes were gathered during the remediation sessions, and translated into detailed summaries at the conclusion of each session. Particular emphasis was given to collecting information on affective traits.

The responses for pre- and post-interviews are given in Table 2.

Table 2 *Number of students demonstrating satisfactory subtraction knowledge in pre- and post-interviews.*

	PRE-INTERVIEW		*POST-INTERVIEW*	
SUBTRACTION KNOWLEDGE	*GROUP 1 Structured reteaching*	*GROUP 2 Old Way/ New Way*	*GROUP 1 Structured reteaching*	*GROUP 2 Old Way/ New Way*
Intuitive				
-solving	2	5	4	7
- creating	5	7	8	8
Concrete	0	2	5	5
Computational	0	0	4	8
Principled-conceptual				
- subtn smaller	2	4	4	6
- addn smaller	3	6	4	7
- renaming	1	5	2	6
- place value	4	6	6	7

Although students were randomly assigned, group 2 had better prior knowledge than group 1, a result that has to be taken into account when interpreting the results. Growth in intuitive, concrete and principled-conceptual knowledge was similar for both groups, although it must be taken into account that treatment time for structured reteaching was 200-minutes, while the treatment time for O/N was 20-minutes. Even with the lesser time, growth in computational knowledge was significantly better for group 2 than for group 1. All students in group 2 (O/N) became satisfactory in computational knowledge.

The observations of the two treatments showed the same marked difference in performance and a marked difference in affective traits. At the start, the mood of the group 1 subjects appeared to be positive (e.g., 'Are we going to have some fun?'). However, this changed when the group 1 students were required to use bundling sticks and MAB to demonstrate the process of attaining the solution rather than the solution itself. They exhibited avoidance behaviour (e.g., they built towers), negative attitudes (e.g., 'I feel a bit daggy doing this!'), and discarded using of materials whenever they were not being directly supervised (e.g., 'I don't need to use blocks. I know how to do this!). The tended to revert to their incorrect pen-and-paper methods. They were easily distracted and complained about what they were required to do (e.g., 'I know how to do these. I want to go back and work with the rest of the class!').

Inspection of the completed worksheets revealed that some students were still performing the algorithm incorrectly, despite apparent positive gains observed during the one-to-one situation with the researcher in previous sessions. From comments and observed body language (such as scowling, groaning, staring out of windows, leaning back on chairs, slowness to respond to required tasks) it was apparent that enthusiasm for these sessions had waned for some students in the group, despite the fact that errors were still being made. Although some students worked steadily at set tasks, they often had to be called back to attention after being distracted by the physical and verbal protestations made by other subjects. The teacher had to work hard to keep the students on task.

At the start of their O/N learning trial, group 2 students were anxious (e.g., 'I can't do these, I always get them wrong!'); and, from their initial comments and body language (slowness to take up the pencil, leaning back in the chair, distancing self from the table), they appeared reluctant to perform the algorithm. By the end, they were ecstatic (e.g., 'Oh, that's good. Now I know how to do it. Good!') and their motivation was high. One student asked whether he could take his work home to show his mother. All students appeared keen to remain together as a group rather than join the bulk of the classroom. An important result was that none of the O/N students reverted to their erroneous procedures.

Discussion and conclusions

Because it models real world situations, structured reteaching is holistic in nature and, therefore, appears to have the potential to effect other knowledge types as well as computational. Because it is symbolic, O/N appears to have too narrow a focus on computational knowledge. The questions are: will structured reteaching match O/N on computational knowledge and will O/N fail to deliver understanding in terms of knowledge types other than computational?

The results of this study appear to support the efficacy of the O/N approach. O/N was much more successful in improving computational knowledge and matched structured reteaching in the other knowledge types. Moreover, the O/N approach was very superior in terms of time, preparation, ease of delivery, motivation, confidence and strength of remediation. Structured reteaching proved to be complicated, effortful and limited in terms of: (1) inability to cater for ability levels; (2) use of materials to produce answers rather than analyse procedures; (3) maintenance of motivation; (4) translation of concrete process to cognitive structure, and to the pen and paper procedure; (5) time and energy requirements; and (6) inability to replace subjects' existing knowledge.

Many factors appeared to affect the success of both remediation methods, but the findings of this study appear to lend support to Lyndon's (1989) argument on the influence of proactive inhibition on remediation. Structured reteaching's use of materials seemed to be in conflict with students' personal, habitual, automatic and erroneous knowledge of the subtraction procedure. As such, several students regressed to their old patterns of error when not monitored by the researcher.

References

Ashlock, R. B. (1986). Error patterns in computation (4th ed.). Ohio: Charles E. Merrill.

Booker, G., Irons, C., & Jones, G. (1980). Fostering arithmetic in the primary school. Canberra: Curriculum Development Centre.

Bourke, S. F. (1980). Numeracy in school: An integrated approach to community expectation, student assessment, and error analysis. Unpublished master's thesis, Monash University, Melbourne.

Brumfield, R., & Moore, B. (1985). Problems with the basic facts may not be the problem. Arithmetic Teacher, 33(3), 17-18.

Covington, M. V. (1985). Strategic thinking and the fear of failure. In J. W. Segal, S. F. Chipman, & R. Glaser (Eds.), Thinking and learning skills: Vol !. Relating instruction to research. Hillsdale, NJ: Lawrence Erlbaum Associates.

Cox, L. S. (1975). Diagnosing and remediating systematic errors in addition and subtraction computations. Arithmetic Teacher. 22 (4), 202-220.

Dole, S. (1992). Unpublished MEd thesis, QUT, Brisbane, Australia.

Ginsburg, H. (1981). The clinical interview in psychological research on mathematical thinking: Aims, rationales, and techniques. For the Learning of Mathematics, 1 (3), 4-11.

James, W. (1890). Principles of psychology. New York: Holt, Rinehart & Winston.

Jones, K., and Charlton, T. (1992). Learning difficulties in primary classrooms: Delivering the whole curriculum. London: Routledge.

Kantowski, M. G. (1978). The teaching experiment and Soviet studies of problem solving. In L. L. Hatfield (Ed.) and D. A. Bradbard (Tech. Ed.). Mathematical problem solving: Papers from a research workshop. ERIC/SMEAC, Ohio State University.

Kephart, N. C. (1960). The slow learner in the classroom. Columbus, Ohio: Charles E. Merrill.

Leinhardt, G. (1988). Getting to know: Tracing student's mathematical knowledge from intuition to competence. Educational Psychologist, 23 (2),119-144.

Lyndon, H. (1989). I did it my way! An introduction to the Old Way/New Way methodology. Australasian Journal of Special Education, 3 (1), 32-37.

Read, G. (1987). Following the progress of children who have received "remedial" tuition. Support for Learning, 2 (1), 52-59.

Underwood, B. J. (1966). Experimental Psychology (2nd ed.). New York: Appleton-Century Crofts.

Wells, D. (1982). The effects of using a manipulative model for initiating regrouping. In M. L. Johnson, L. A. Raivek, J. Pitt, N. Wilson, & D. Wells (Eds.), Systematic approaches to remediation of mathematical difficulties. Proceedings of the Ninth Annual Conference of the Research Council for Diagnostic Prescriptive Mathematics.

Wilson, N. (1982). A remedial procedure for a specific multiplication error. In M. L. Johnson, L. A. Raivek, J. Pitt, N. Wilson, & D. Wells (Eds.), Systematic approaches to remediation of mathematical difficulties. Proceedings of the Ninth Annual Conference on the Research Council for Diagnostic Prescriptive Mathematics.